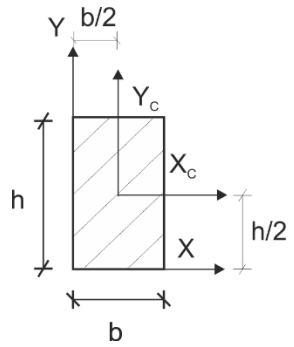
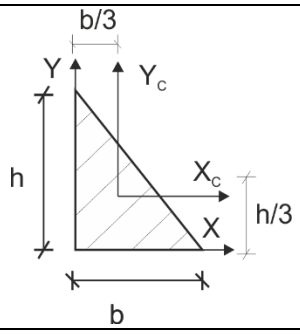
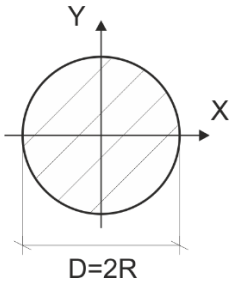
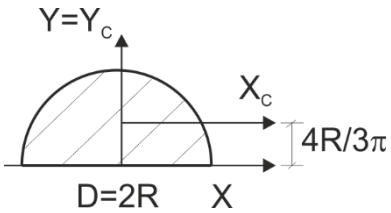
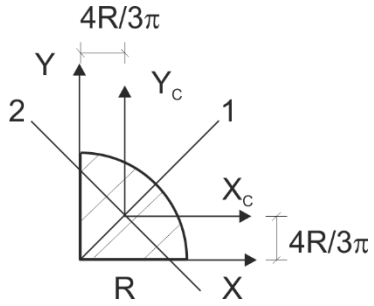
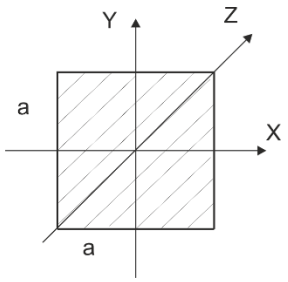
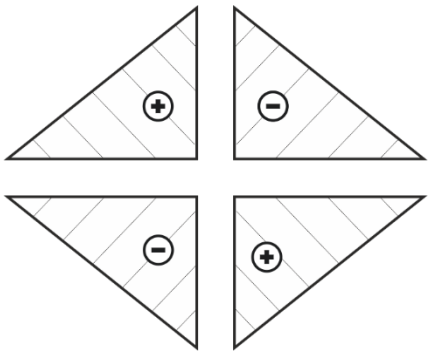
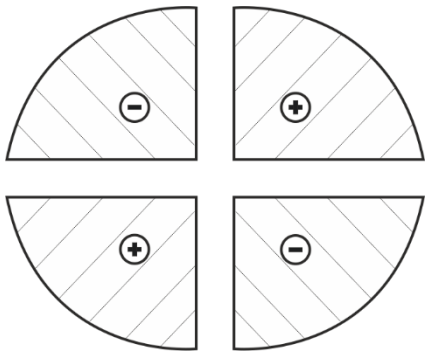
Momenty bezwładności figur płaskich

Figura	*Osi XcYc*	*Osi XY*
	$J_{X_c} = \frac{b \cdot h^3}{12}$ $J_{Y_c} = \frac{h \cdot b^3}{12}$ $J_{X_cY_c} = 0$	$J_X = \frac{b \cdot h^3}{3}$ $J_Y = \frac{h \cdot b^3}{3}$ $J_{XY} = \frac{b^2h^2}{4}$
	$J_{X_c} = \frac{b \cdot h^3}{36}$ $J_{Y_c} = \frac{h \cdot b^3}{36}$ $J_{X_cY_c} = -\frac{b^2h^2}{72}$	$J_X = \frac{b \cdot h^3}{12}$ $J_Y = \frac{h \cdot b^3}{12}$ $J_{XY} = \frac{b^2h^2}{24}$
	$J_{X_c} = \frac{\pi \cdot D^4}{64} = \frac{\pi R^4}{4}$ $J_{Y_c} = \frac{\pi \cdot D^4}{64} = \frac{\pi R^4}{4}$ $J_{X_cY_c} = 0$	
	$J_{X_c} \approx 0{,}00686D^4 \approx 0{,}1098R^4$ $J_{X_cY_c} = 0$	$J_X = \frac{\pi D^4}{128} = \frac{\pi R^4}{8}$ $J_Y = \frac{\pi D^4}{128} = \frac{\pi R^4}{8}$ $J_{XY} = 0$
	$J_{X_c} \approx 0{,}0549R^4$ $J_{X_cY_c} = -0{,}0165R^4$	$J_X = \frac{\pi D^4}{256} = \frac{\pi R^4}{16}$ $J_Y = \frac{\pi D^4}{256} = \frac{\pi R^4}{16}$ $J_{XY} = \frac{R^4}{8}$

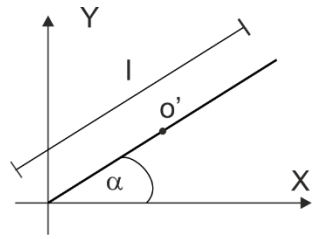
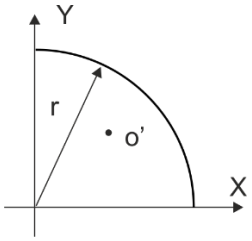
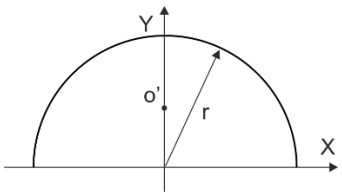
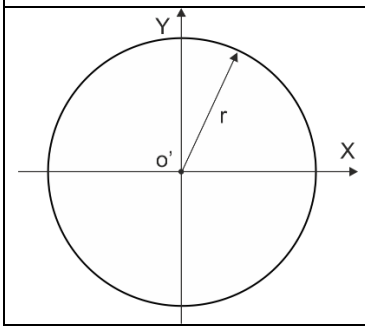
	$J_{X_c} = J_{Y_c} = J_{Z_c} = \frac{a^4}{12}$	

Znakowanie momentów dewiacja





Momenty bezwładności i dewiacji krzywych płaskich (o gęstości = 1)

Krzywa	Środek masy	*Osi XcYc*	*Osi XY*
	$x' = \frac{l}{2}\cos\alpha$ $y' = \frac{l}{2}\cos\alpha$	$J_{X_C} = \frac{l^3}{12}\sin^2\alpha$ $J_{Y_C} = \frac{l^3}{12}\cos^2\alpha$ $J_{X_CY_C} = \frac{l^3}{12}\sin^2\alpha\cdot\cos^2\alpha$	$J_X = \frac{l^3}{3}\sin^2\alpha$ $J_Y = \frac{l^3}{3}\cos^2\alpha$ $J_{XY} = \frac{l^3}{3}\sin^2\alpha\cdot\cos^2\alpha$
	$x' = \frac{2\cdot r}{\pi}$ $y' = \frac{2\cdot r}{\pi}$	$J_{X_C} = J_{Y_C} = r^3\left(\frac{\pi}{4} - \frac{2}{\pi}\right)$ $J_{X_CY_C} = r^3\left(\frac{1}{2} - \frac{2}{\pi}\right)$	$J_X = J_Y = \frac{\pi\cdot r^3}{4}$ $J_{XY} = \frac{r^3}{2}$
	$x' = 0$ $y' = \frac{2\cdot r}{\pi}$	$J_{X_C} = 2\cdot r^3\left(\frac{\pi}{4} - \frac{2}{\pi}\right)$ $J_{Y_C} = \frac{\pi\cdot r^3}{2}$ $J_{X_CY_C} = 0$	$J_X = J_Y = \frac{\pi\cdot r^3}{2}$ $J_{XY} = 0$
	$x' = 0$ $y' = 0$	$J_{X_C} = J_{Y_C} = \pi\cdot r^3$ $J_{X_CY_C} = 0$	$J_X = J_Y = \pi\cdot r^3$ $J_{XY} = 0$

Wzór Steinera

$$I_{yC} = I_{y1} + A_1(z_1 - z_c)^2 \pm \left[I_{y2} + A_2(z_2 - z_c)^2\right] \pm \cdots$$

$$I_{zC} = I_{z1} + A_1(y_1 - y_c)^2 \pm [I_{z2} + A_2(y_2 - y_c)^2] \pm \cdots$$

$$D_C = D_1 + A_1(y_1 - y_c)(z_1 - z_c) \pm [D_2 + A_2(y_2 - y_c)(z_2 - z_c)] \pm \cdots$$

Momenty bezwładności główne centralne

$$I_1 = \frac{I_{yC} + I_{zC}}{2} + \sqrt{\left(\frac{I_{yC} - I_{zC}}{2}\right)^2 + D_C^2}$$

$$I_2 = \frac{I_{yC} + I_{zC}}{2} - \sqrt{\left(\frac{I_{yC} - I_{zC}}{2}\right)^2 + D_C^2}$$

Kąt obrotu osi

$$tg\varphi_1 = \frac{D_C}{I_{zC} - I_1}$$

$$tg\varphi_2 = \frac{D_C}{I_{zC} - I_2}$$